

NEW PHOTON PROPAGATORS IN QUANTUM ELECTRODYNAMICS

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A Lagrangian for quantum electrodynamics is found which makes it explicit that the photon mass is eventually set to zero in the physical part on observational ground. It remains possible to obtain a counterterm Lagrangian where the only non-gauge-invariant term is proportional to the squared divergence of the potential, while the photon propagator in momentum space falls off like k^{-2} at large k , which indeed agrees with perturbative renormalizability. The resulting radiative corrections to the Coulomb potential in QED are also shown to be gauge-independent. A fundamental role of the space of 4-vectors with components given by 4×4 matrices is therefore suggested by our scheme, where such matrices can be used to define a single gauge-averaging functional in the path integral.

1 Introduction

Our research on quantum electrodynamics (hereafter QED) has been motivated by the need to understand how to quantize gauge theories of fundamental interactions in case the Higgs boson were to remain elusive. Although we have failed in this respect, we have found a number of field-theoretical properties which seem to be of interest and are now described, relying upon Ref. [1].

To begin, we recall that gauge-invariant Lagrangians are naturally considered to generate mass via the Higgs mechanism, if fundamental scalar fields are taken to exist. But gauge-invariant Lagrangians suffer from a “degeneracy” in that they lead to non-invertible operators on gauge fields. At about the same time when Higgs was elaborating his model, it became clear thanks to Feynman, DeWitt, Faddeev and Popov (and various other authors after them) that the appropriate Lagrangian contains actually (at least) 3 ingredients:

- (i) A gauge-invariant part \mathcal{L}_{inv} ;
- (ii) A term called *gauge-breaking*, *gauge-fixing* or *gauge-averaging*;
- (iii) Contribution of ghost fields [they have classical roots; just think of the

need to preserve the supplementary condition under gauge transformations, which leads to a gauge function ruled by a differential operator].

The resulting physical predictions are independent of the particular supplementary condition and of any choice for the matrix of gauge parameters.

Thus, a gauge-invariant Lagrangian \mathcal{L}_{inv} with scalars leads to the Higgs mechanism for mass generation. But if no fundamental scalar field exists, we face the problem of studying a Becchi–Rouet–Stora–Tyutin-invariant Lagrangian and understanding under which conditions this is compatible with mass terms (which is not the same as generating mass!).

2 New gauges for Maxwell and QED

Our first basic remark is that

$$A_\mu A^\mu = g^{\mu\nu} A_\mu A_\nu = \frac{1}{4} \text{Tr}(\gamma^\mu \gamma^\nu) A_\mu A_\nu. \quad (1)$$

We have therefore looked, in a first moment, for a linear gauge combining the effect of Lorenz gauge [2] and γ -matrices. However, one cannot simply add the derivatives of A_μ in the Lorenz gauge and γ^μ terms, since the latter are 4-vectors with components given by 4×4 matrices. The only well defined operation on such objects is the one giving rise to the matrix

$$\Phi_i^j(A) \equiv (\delta_i^j \partial^\mu + \beta (\gamma^\mu)_i^j) A_\mu(x), \quad (2)$$

where the parameter β is introduced to ensure that all terms in Φ_i^j have the same dimension (i.e. β has dimension $[\text{length}]^{-1}$). There is only one coefficient, β , since only one potential A_μ is available for contraction with γ^μ in the Abelian case.

The resulting gauge-averaging term in the Lagrangian is taken to be

$$\mathcal{L}_{G.A.} = \frac{\Phi^2(A)}{2\alpha} = \frac{1}{2\alpha} \Phi_i^j(A) \Omega_j^k \Phi_k^i(A), \quad (3)$$

where we have defined the symmetric matrix

$$\Omega_j^k \equiv \frac{1}{4} \delta_j^k. \quad (4)$$

Therefore ($g^{\mu\nu} = -\frac{1}{4} \text{Tr}(\gamma^\mu \gamma^\nu)$ and $\beta \rightarrow \pm i\beta$ in Euclidean theory)

$$\begin{aligned} \Phi_i^j(A) \Omega_j^k \Phi_k^i(A) &= \frac{1}{4} [4(\partial^\mu A_\mu)(\partial^\nu A_\nu) + \beta(\partial^\mu A_\mu)(\gamma^\nu)_i^j A_\nu + \beta(\gamma^\mu)_i^j A_\mu \partial^\nu A_\nu \\ &+ \beta^2(\gamma^\mu)_i^j (\gamma^\nu)_j^i A_\mu A_\nu] = (\partial^\mu A_\mu)(\partial^\nu A_\nu) + \beta^2 A_\mu A^\mu. \end{aligned} \quad (5)$$

It should be stressed that the matrix $\Phi_i^j(A)$ is a tool to express in a concise and elegant form the gauge-averaging term in the full action, but *our gauge-averaging functional for QED is not a matrix* and is equal to

$$\Phi(A) \equiv \left(\Phi_i^j(A) \Omega_j^k \Phi_k^i(A) \right)^{\frac{1}{2}}. \quad (6)$$

One cannot regard $\Phi_i^j(A)$ itself as a gauge-averaging functional, since otherwise one would get 16 supplementary conditions which are totally extraneous to the quantum (as well as classical) theory. One can however say that the matrix (2) acts as a “potential” for the gauge-averaging functional, in that the definition (6) can be given. The price to be paid is lack of a simple formula for the ghost operator \mathcal{P} , whose action is given by

$$\mathcal{P} : \varepsilon \rightarrow - \left[\frac{(\partial^\nu A_\nu)}{\Phi(A)} \partial^\mu \partial_\mu + \frac{\beta^2 A^\mu}{\Phi(A)} \partial_\mu \right] \varepsilon, \quad (7)$$

which however does not spoil the occurrence of $A_\mu A^\mu$ terms in the full action functional. Thus, the starting point remains the well known property that only one gauge-fixing condition ($\Phi(A) = \zeta$) is needed, and we choose $\Phi(A)$ in the non-linear form (6).

3 Bare photon propagator

The photon propagator is obtained by first considering the gauge-field operator $P_{\mu\nu}$ from the path integral, then taking its symbol $\sigma(P_{\mu\nu}) \equiv \sigma_{\mu\nu}$, and eventually inverting σ to find $\tilde{\sigma}$ such that

$$\sigma_{\mu\nu} \tilde{\sigma}^{\nu\lambda} = \delta_\mu^\lambda. \quad (8)$$

This leads to

$$\Delta^{\mu\nu}(x, y) = \int_\zeta \frac{d^4 k}{(2\pi)^4} \tilde{\sigma}^{\mu\nu}(k) e^{ik \cdot (x-y)}. \quad (9)$$

We find, by virtue of our gauge-averaging term,

$$\mathcal{L} = \partial^\mu \rho_\mu + \frac{1}{2} A^\mu P_{\mu\nu} A^\nu, \quad (10)$$

where

$$\rho_\mu \equiv -\frac{1}{2} A_\nu \partial^\nu A_\mu + \frac{1}{2} A_\nu \partial_\mu A^\nu + \frac{1}{2\alpha} A_\mu \partial^\nu A_\nu, \quad (11)$$

$$P_{\mu\nu} \equiv g_{\mu\nu} \left[-\square + \frac{\beta^2}{\alpha} \right] + \left(1 - \frac{1}{\alpha} \right) \partial_\mu \partial_\nu. \quad (12)$$

Of course, $\text{div} \rho$ does not affect the field equations, and $P_{\mu\nu}$ depends on gauge parameters. In particular, we here choose for simplicity:

$$P_{\mu\nu}(\alpha = 1) = g_{\mu\nu} \left(-\square + \beta^2 \right), \quad (13)$$

which implies

$$\sigma(P_{\mu\nu}(\alpha = 1)) = (k^2 + \beta^2)g_{\mu\nu}, \quad (14)$$

with Euclidean photon propagator

$$\Delta_E^{\mu\nu}(x, y) = \int_{\Gamma} \frac{d^4 k}{(2\pi)^4} \frac{g^{\mu\nu}}{(k^2 + \beta^2)} e^{ik \cdot (x-y)}. \quad (15)$$

Integration along the real axis avoids poles of the integrand, and the interpretation of the massive term will become clear in the following sections.

4 Perturbative renormalization

According to the perturbative renormalization programme, we now distinguish between bare quantities, denoted by the B subscript, and physical quantities (which do not carry any subscript). This is done for all fields, physical parameters and gauge parameters by assuming that multiplicative renormalizability still holds with our gauge (6), so that we can exploit the relations

$$(A_\mu)_B = \sqrt{z_A} A_\mu, \implies F_B^{\mu\nu} = \partial^\mu A_B^\nu - \partial^\nu A_B^\mu, \quad (16)$$

$$\psi_B = \sqrt{z_\psi} \psi, \quad (17)$$

$$m_B = \frac{z_m}{z_\psi} m, \quad (18)$$

$$e_B = \frac{z_e}{z_\psi \sqrt{z_A}} e, \quad (19)$$

$$\alpha_B = \frac{z_A}{z_\alpha} \alpha, \quad (20)$$

$$\beta_B = \rho \beta, \quad (21)$$

where the ρ coefficient will be fixed shortly. Hence we find

$$\mathcal{L} - \mathcal{L}_{\text{gh}} = \mathcal{L}_{\text{ph}} + \mathcal{L}_{\text{ct}}, \quad (22)$$

where the physical part reads

$$\begin{aligned}\mathcal{L}_{\text{ph}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\gamma^\mu\partial_\mu\psi - e\bar{\psi}\gamma^\mu A_\mu\psi - m\bar{\psi}\psi \\ &- \frac{1}{2\alpha}(\partial^\mu A_\mu)^2 - \frac{\beta^2}{2\alpha}A_\mu A^\mu,\end{aligned}\quad (23)$$

with

$$\frac{\beta^2}{\alpha} \equiv m_\gamma^2, \quad (24)$$

while the part involving counterterms is given by

$$\begin{aligned}\mathcal{L}_{\text{ct}} &= -\frac{1}{4}(z_A - 1)F_{\mu\nu}F^{\mu\nu} + (z_\psi - 1)\bar{\psi}i\gamma^\mu\partial_\mu\psi \\ &- (z_e - 1)e\bar{\psi}\gamma^\mu A_\mu\psi - (z_m - 1)m\bar{\psi}\psi \\ &- \frac{1}{2\alpha}(z_\alpha - 1)(\partial^\mu A_\mu)^2 - \frac{\beta^2}{2\alpha}(\rho^2 z_\alpha - 1)A_\mu A^\mu.\end{aligned}\quad (25)$$

Note that, if

$$\rho = \frac{1}{\sqrt{z_\alpha}}, \quad (26)$$

the counterterm Lagrangian reduces to the familiar form in the Lorenz gauge, and the renormalization of β is not independent of the renormalization of α , in agreement with (24). At this stage we deal with a freely specifiable gauge parameter, i.e. α , and with the physical mass parameter m_γ .

5 Radiative corrections in QED

Since

$$\frac{\beta_B^2}{\alpha_B} = \frac{\rho^2 \beta^2}{\frac{z_A}{z_\alpha} \alpha} = \frac{m_\gamma^2}{z_A} \equiv \widetilde{m}_\gamma^2, \quad (27)$$

we find in the bare theory

$$\sigma_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)(k^2 + \widetilde{m}_\gamma^2) + \frac{k_\mu k_\nu}{k^2} \frac{1}{\alpha_B}(k^2 + \alpha_B \widetilde{m}_\gamma^2), \quad (28)$$

with inverse

$$\tilde{\sigma}^{\mu\nu}(k) = \frac{g^{\mu\nu}}{(k^2 + \widetilde{m}_\gamma^2)} + \frac{(\alpha_B - 1)k^\mu k^\nu}{(k^2 + \alpha_B \widetilde{m}_\gamma^2)(k^2 + \widetilde{m}_\gamma^2)}. \quad (29)$$

Note that $\tilde{\sigma}^{\mu\nu}$ falls off like k^{-2} at large k , in agreement with perturbative renormalizability.

In the renormalized theory one has on general ground

$$\Sigma_{\mu\nu}(k) = g_{\mu\nu}u_1(k^2) + k_\mu k_\nu u_2(k^2), \quad (30)$$

$$\tilde{\Sigma}^{\mu\nu}(k) = g^{\mu\nu}d_1(k^2) + k^\mu k^\nu d_2(k^2), \quad (31)$$

while diagrammatic analysis shows that

$$\tilde{\Sigma}^{\mu\nu}(k) = \tilde{\sigma}^{\mu\nu}(k) + \tilde{\sigma}^{\mu\lambda}\Pi_{\lambda\rho}\tilde{\Sigma}^{\rho\nu}(k), \quad (32)$$

with $\Pi_{\lambda\rho}$ the polarization tensor, given by

$$\begin{aligned} \Pi_{\mu\nu}(k) &= \sigma_{\mu\nu}(k) - \Sigma_{\mu\nu}(k) \\ &= g_{\mu\nu} \left(k^2 + \widetilde{m}_\gamma^2 - u_1 \right) + k_\mu k_\nu \left(\frac{1}{\alpha_B} - 1 - u_2 \right) \\ &= g_{\mu\nu}a_1(k^2) + k_\mu k_\nu a_2(k^2). \end{aligned} \quad (33)$$

Current conservation implies that $k^\mu \Pi_{\mu\nu} = 0$, which leads to

$$a_1 = -k^2 a_2, \quad (34)$$

$$u_1 + k^2 u_2 = \frac{1}{\alpha_B} (k^2 + \alpha_B \widetilde{m}_\gamma^2), \quad (35)$$

and hence

$$\Pi_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \left(k^2 + \widetilde{m}_\gamma^2 - u_1 \right). \quad (36)$$

This is in gauge-independent form, as expected.

For example, we may set

$$u_1 = \widetilde{m}_\gamma^2 + f(k^2), \quad (37)$$

which implies

$$u_2 = \frac{1}{\alpha_B} - \frac{f(k^2)}{k^2}. \quad (38)$$

Therefore

$$\Sigma_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \left(f(k^2) + \widetilde{m}_\gamma^2 \right) + \frac{k_\mu k_\nu}{k^2} \frac{1}{\alpha_B} (k^2 + \alpha_B \widetilde{m}_\gamma^2). \quad (39)$$

Thus, the coefficient of the longitudinal part $\frac{k_\mu k_\nu}{k^2}$ is the same in the bare as well as in the full theory, while the coefficients of the transverse part $g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$ depend on α_B and β_B in such a way that the difference $\sigma_{\mu\nu} - \Sigma_{\mu\nu}$ is indeed gauge-independent:

$$\Pi_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) (k^2 - f(k^2)). \quad (40)$$

The renormalized photon propagator in momentum space is the inverse of $\Sigma_{\mu\nu}(k)$, i.e.

$$\tilde{\Sigma}^{\mu\nu}(k) = \frac{g^{\mu\nu}}{(f(k^2) + \widetilde{m}_\gamma^2)} + \frac{(\alpha_B \frac{f(k^2)}{k^2} - 1) k^\mu k^\nu}{(k^2 + \alpha_B \widetilde{m}_\gamma^2) (f(k^2) + \widetilde{m}_\gamma^2)}. \quad (41)$$

As an application, we consider radiative corrections to Coulomb's law:

$$\mathcal{A}^0 = A^0 + \tilde{\Sigma}^{0\rho} \Pi_{\rho\lambda} A^\lambda, \quad (42)$$

where

$$A^0 = \text{const.} \frac{q}{k^2}, \quad (43)$$

while

$$\tilde{\Sigma}^{0\rho} \Pi_{\rho\lambda} = \left(\delta_\lambda^0 - \frac{k^0 k_\lambda}{k^2} \right) \frac{(k^2 - f(k^2))}{(f(k^2) + \widetilde{m}_\gamma^2)}, \quad (44)$$

since $k^0 k^\rho d_2(k^2) \Pi_{\rho\lambda} = 0$. In other words, we find that the renormalized potential \mathcal{A}^0 depends on gauge parameters α, β not separately, which would have led to unavoidable gauge dependence (since $\beta = m_\gamma \sqrt{\alpha}$), but only through the product $\frac{1}{z_A} \frac{\beta^2}{\alpha}$. The latter is proportional to the photon mass parameter m_γ^2 in the physical Lagrangian of perturbative renormalization. Thus, *the resulting short-range potential only depends on a mass parameter in the physical Lagrangian and is therefore, with the above understanding, gauge independent*. Note that the classical long-range part $\frac{q}{k^2}$ resulting from A^0 is still present, and eventually our \widetilde{m}_γ is set to zero on observational ground.

Note also that, in the light of previous remarks, the most general form of u_1 is

$$u_1 = u_1 \left(k^2; \frac{\beta_B^2}{\alpha_B} \right) = u_1(k^2; \widetilde{m}_\gamma^2), \quad (45)$$

leading to

$$d_1 = \frac{1}{u_1(k^2; \widetilde{m}_\gamma^2)}, \quad (46)$$

and

$$d_2 = \frac{1}{k^2} \left[\frac{\alpha_B}{(k^2 + \alpha_B \widetilde{m}_\gamma^2)} - \frac{1}{u_1(k^2; \widetilde{m}_\gamma^2)} \right]. \quad (47)$$

Once more, only d_2 is gauge-dependent, since its first term depends on α_B .

6 Results and open problems

We have not truly generated mass for the photon, but we have developed tools for a more thorough treatment of its massless nature. More precisely, our original results are as follows.

- (i) Derivation of the gauge-averaging functional (6) for QED.
- (ii) New photon propagators in quantum electrodynamics, with m_γ as an explicit mass parameter in the physical Lagrangian.
- (iii) Renormalization of the gauge parameter β in such a way that the counterterm Lagrangian has only one term which is not gauge-invariant.
- (iv) Renormalized photon propagator in our gauges, and proof of gauge independence of the associated short-range potential, adding evidence in favour of our model being physically relevant.

Objections (O) can be raised and answers (A) can be given along the following lines:

(O1) Precisely in the Abelian case, the Higgs–Kibble (HK) model has a mass term which is cohomologically non-trivial and hence it has physical content. Such a property is hidden by your model.

(A1) The HK model relies upon fundamental scalar fields, whereas we have tried to understand what happens if such scalar fields do not exist.

(O2) The approach presented is, eventually, very phenomenological, in that suitable combinations of gauge parameters are used to fit the experimental data.

(A2) Yes indeed, without fundamental scalar fields there remains a fundamental procedure, i.e. construction of a Lagrangian leading to invertible operators on the potentials, but then we end up fixing physical parameters on observational ground. It might be acceptable to the extent that we are satisfied with

perturbative renormalization.

Note also that in the massive QED model, which is ruled by the field equations

$$(i\gamma^\mu \partial_\mu - MI)\psi = eA^\mu \gamma_\mu \psi, \quad (48)$$

$$\partial^\mu F_{\mu\nu} + m^2 A_\nu = -e\bar{\psi} \gamma_\nu \psi, \quad (49)$$

the photon propagator is given by

$$\Delta^{\mu\nu}(x, y) = \int \frac{d^4 k}{(2\pi)^4} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m^2} \right) \frac{-i}{(k^2 - m^2 + i\varepsilon)} e^{ik \cdot (x-y)}. \quad (50)$$

Its integrand is constant at large k , and this leads to a non-renormalizable theory, in which the divergence of a Feynman diagram increases with the number of internal photon lines. Such unpleasant features are not shared by our photon propagator (in agreement with our analysis of perturbative renormalizability), which keeps the standard k^{-2} behaviour at large k .

The main open problem is as follows. On the one hand, no gauge-averaging is needed in the path integral for fermionic fields. Thus, by construction, our approach does not generate masses for fermions, and hence does not provide an alternative to the Higgs mechanism in the standard model. On the other hand, we end up by putting the emphasis on the space of 4-vectors with components given by 4×4 matrices, which is a natural structure for theories incorporating fermions.

As far as we can see, further research topics are suggested by our approach, in particular: (i) How to prove explicitly perturbative renormalizability in the non-Abelian case in our broader framework; (ii) Possible occurrence of Gribov ambiguities in the non-perturbative formulation of non-Abelian theories; (iii) Evaluation of mass terms for ghost fields; (iv) Equations for Green functions in QED and quantum Yang–Mills along the lines of Gribov [3]. His idea was to formulate equations for Green's functions and vertices in a form that does not contain any divergences, so that solutions of these equations will be able to account for both perturbative and non-perturbative phenomena.

[1] G. Esposito, *Found. Phys.* **32** (2002) 1459.

[2] L. Lorenz, *Phil. Mag.* **34** (1867) 287.

[3] V.N. Gribov, *Eur. Phys. J.* **C 10** (1999) 71.